extended further. However, at low densities and high speeds additional complications of nonequilibrium phenomenon occurs, and the results derived previously can then be altered to a substantial degree (see Ref. 4).

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Magnetohydrodynamic Effect on Open-Channel Flow

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Introduction

The behavior of a conducting fluid, flowing in the absence of an electric field in an open channel under the influence of gravity and an applied vertical magnetic field, was investigated theoretically and experimentally.

As shown in Fig. 1, the mercury enters the channel with an x-directed uniform velocity (V_0). The effect of the magnetic field is to induce an electric field and a current (j) in the negative z direction. The current returns outside the channel through an artificial perfect conductor. Since the current is transverse to the magnetic field (B), a $j \times B$ body force is induced in the negative x direction. The approximate values of the independent parameters for the cases tested are as follows: velocity (V_0) = 1 cm/sec, channel height (Y_0) = 7.5 cm, channel length (L_0) = 15 cm, and magnetic field (B) = 10 kgauss.

These values of the independent parameters are used with mercury as the conducting fluid. The following assumptions are reasonable: first, all of the induced magnetic fields are neglected (the magnetic Reynolds number is of the order of 10^{-3}); second, the flow is assumed inviscid (the Reynolds numbers under consideration are of the order of 10^4); third,

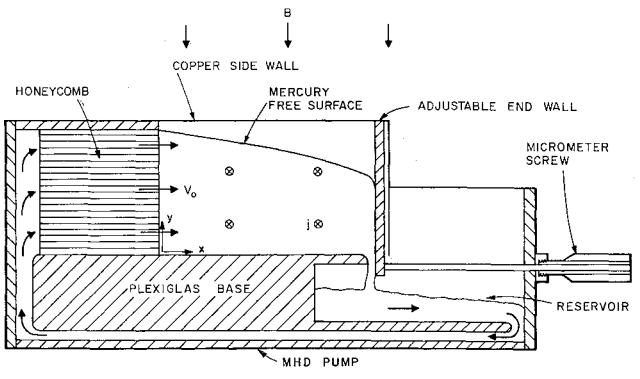

Fig. 1 Sectional schematic of flow system.

Received November 5, 1964; revision received March 8, 1965. The author wishes to thank W. H. Heiser, Assistant Professor of Mechanical Engineering at Massachusetts Institute of Technology, for the many helpful discussions during the course of this work.

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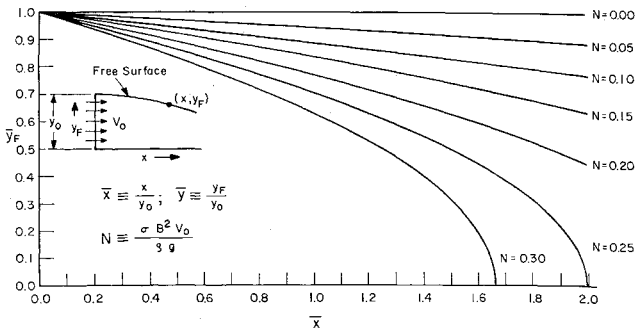

Fig. 2 Theoretical curves of free surface.

since the displacement thickness is small as compared to the characteristic channel dimensions, the influence of the boundary layer is neglected; fourth, inertia terms are neglected, in that $V_0^2/y_0 g$ is of the order of 10^{-4}.

Theoretical Analysis

The linearized momentum equation for the steady flow of an inviscid and incompressible fluid is, in this case,

$$\nabla P + \sigma u B^2 i_x + \rho g i_y = 0 \tag{1}$$

where P is the pressure, σ the fluid conductivity, u the x component of velocity, ρ the mass density, g the acceleration of gravity, and i_x and i_y are unit vectors in the x and y directions, respectively.

By taking the curl of Eq. (1), it readily is seen that $\partial u/\partial y$ must vanish. Hence, the continuity equation can be written as

$$u y_F = V_0 y_0 \tag{2}$$

where y_F is the vertical distance from the bottom of the channel to the free surface. The momentum equation written along a streamline is

$$(1/\rho) dP + (\sigma u B^2/\rho)(dx)_s + g(dy)_s = 0 \tag{3}$$

where $(dx)_s$ and $(dy)_s$ are such that $(dy)_s/(dx)_s$ is the slope of the streamline. Writing Eq. (3) for the free surface streamline and substituting for u from Eq. (2) yields

$$(dy)_F/(dx)_F = -(\alpha B^2 V_0 y_0/\rho y_F) \tag{4}$$

where $(dy)_F/(dx)_F$ is the slope of the free surface streamline. Integration of Eq. (4) from 0, y_0 to some arbitrary point x, y_F, and subsequent normalization yields the equation for the free surface. Thus

$$\bar{y}_F = (1 - 2N\bar{x})^{1/2} \tag{5}$$

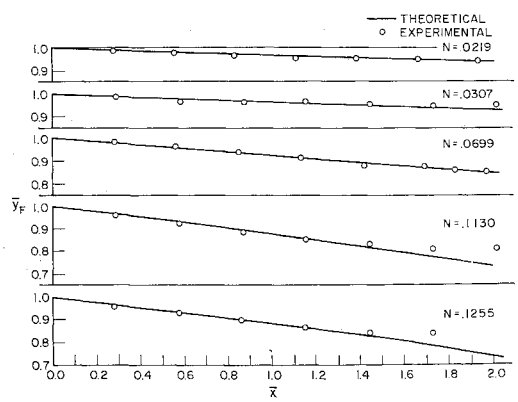

Fig. 3 Comparison of experimental and theoretical free surfaces.

where $\bar{y}_F = y_F/y_0$, $\bar{x} = x/y_0$, and $N = \sigma B^2 V_0/\rho g$. Equation (5) is presented graphically in Fig. 2 for various values of N. It is evident that $\bar{y}_F$ goes to zero as $2N\bar{x}$ approaches unity, which is physically untenable. Indeed, for small values of $\bar{y}_F$, the inertia terms cannot be neglected, and Eq. (5) becomes invalid.

Results

The experimental results were obtained photographically, using the flow system shown schematically in Fig. 1. Figure 3 shows the experimentally determined coordinates of the free surface superimposed on the theoretical curves for corresponding values of N. The agreement is good, except near the exit, for curves characterized by large values of N. This discrepancy is probably the result of secondary flows in the form of strong corner vortices observed in the neighborhood of the exit.

Conclusions

The techniques that were successfully employed to solve the foregoing problem could be used to simplify other similar but more difficult magnetohydrodynamics (MHD) problems. For example, the fact that the x component of velocity is uniform with y is also true for the problem of the unsteady emptying of a fluid from a tank or the extension of the foregoing problem to cylindrical geometry.

Mass Transfer Cooling in Laminar Hypersonic Cavity Flow

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Nomenclature

D = cavity depth
H = local heat-transfer coefficient
$\bar{H}$ = average heat-transfer coefficient
L = cavity length
M = Mach number
$\dot{M}$ = gas injection rate
Q = local heat-transfer rate
$\bar{Q}$ = average heat-transfer rate
Re_∞ = freestream Reynolds number
X = distance measured from upstream cavity wall

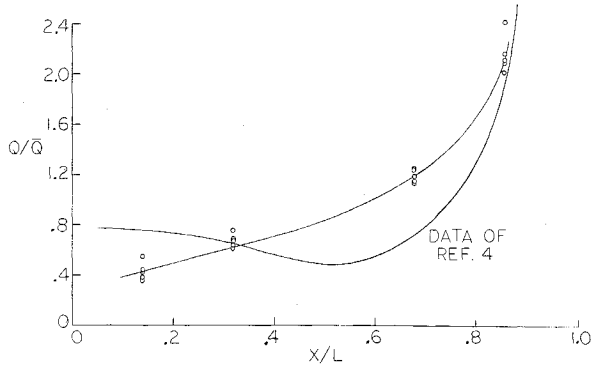

Fig. 1 Heat-transfer distribution with no injection.

Received February 8, 1965.

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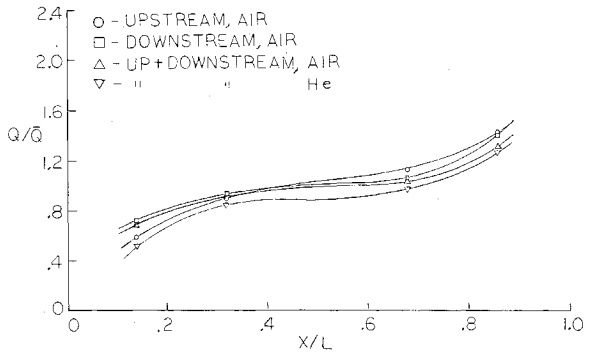

Fig. 2 Heat-transfer distribution with gas injection.

Analysis[1] has indicated that the heat-transfer rate through a separated laminar boundary layer would be reduced to 0.56 of that which would occur through the corresponding attached laminar boundary layer. This has been confirmed by Charwat, Dewey, et al.[2] who determined experimentally the heat-transfer distribution existing on the floor of a two-dimensional rectangular cavity in a separated supersonic stream. They found that, when the boundary-layer thickness at the upstream edge of the cavity was small in comparison to the depth of the cavity, the average heat-transfer rate to the floor of the cavity was reduced by a factor of approximately 2. Furey[3] obtained similar experimental results. Heat transfer in axisymmetric separated regions was experimentally determined by Larson[4] and Nicoll,[5] and again agreement with Ref. 1 was obtained.

Chapman[1] further predicted that the introduction of a coolant gas into a separated region would have a powerful effect in reducing the rate of heat transfer through the region. In fact, he predicted that a moderate quantity of gas injected into the separated region would reduce the heat transfer to zero. Presented here are experimental results showing the accuracy of this prediction.

Heat transfer with gas injection was measured for a rectangular cavity on a 20° total angle cone at a Mach number of 6.86 in air. The cavity flow was laminar (Re_∞/ft $\doteq 2 \times 10^6$) and "open" ($L/D = 6.7$). All experiments were conducted in the 3-in. hypersonic wind tunnel of IIT Research Institute which is described in Ref. 6.

Heat-transfer measurements were made using a transient calorimeter technique. The cavity floor was instrumented with five equally spaced copper-ring calorimeters. Means for injecting gas into the cavity were provided by two manifolds, which formed the upstream and downstream walls of the cavity.

In Fig. 1, the results are shown for the heat-transfer distribution in the cavity with no injection, which are compared to that obtained by Larson[4] for a rectangular cavity on a cylinder at $M = 4$. The average heat-transfer rate was found to be 50% of the heat-transfer rate of a corresponding

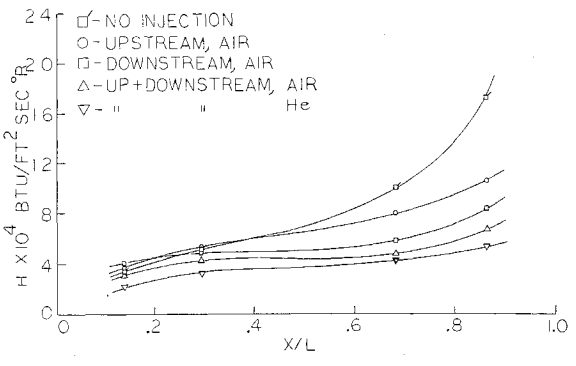

Fig. 3 Distribution of heat-transfer coefficient.